Maths

A whole school approach to problem solving

At Woodside, we have a whole school approach to problem solving. We follow the White Rose Maths programme, which gives all pupils opportunities to solve problems, reason, conjecture, prove and generalise. With every unit of work, White Rose also provides teachers with an additional bank of problem solving and reasoning resources which all children are familiar with using.

As a school, we have identified the most useful problem solving strategies that pupils will learn. These have been explored in depth with staff and everyone has a clear understanding of what they are and how to teach them. The White Rose curriculum ensures that children have access to a wide range of different types of problems each term. Our teachers also supplement the White Rose curriculum with additional resources that require children to reason and problem solve.

We know that is important to explicitly teach children the different types of problem solving strategies that they can draw on. In lessons, pupils tackle problem solving tasks that require them to not only work out the answer, but also work out the process for finding that answer.

Across the school, our pupils also consistently use ‘RUCSAC’ to help them solve problems. This helps the pupils to identify and separate out the important information in a problem. We also understand the importance of recall and automaticity, with children having acquired the facts, concepts and methods required to tackle a problem successfully. Pupils are familiar with different types of problems and a range of strategies to draw on and these are displayed in all classrooms on the Maths working walls.

Different problem-solving skills and strategies

- Trial and improvement
- Working systematically
- Finding all possibilities
- Drawing a table
- Pattern spotting
- Working backwards
- Reasoning logically
- Visualising
- Conjecturing (forming an opinion on something without all of the information)
- Generalising
- Proving
- Word Problems
- Drawing a bar model

Read

Read the question.

What is the important information?

Understand

Understand the question. What do you need to do? Underline important information.

Choose

Choose the operations/ calculations/ method/ strategy.

Solve

Solve the problem.

Answer

Answer the question.

What were you meant to find out?

Check

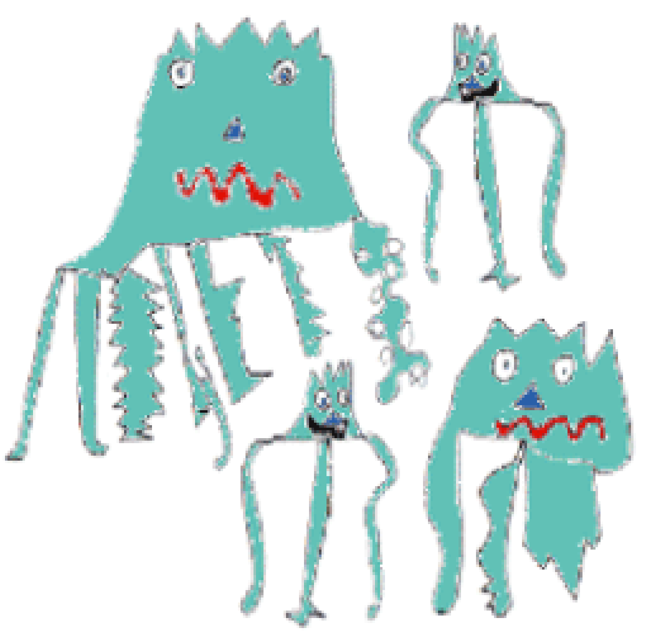
Check your answer using checking strategies such as the inverse/ estimation.

Trial and improvement

This is perhaps an under-valued skill. Children can be reluctant to use trial and improvement as they sometimes feel they are only using it because they do not know the 'right' way to solve the problem in hand. In reality, trial and improvement involves trying something out, which will always give more insight into the context and therefore gives the solver a better idea of what to try next. Trial and improvement is often the start of working systematically.

The Zios and Zepts

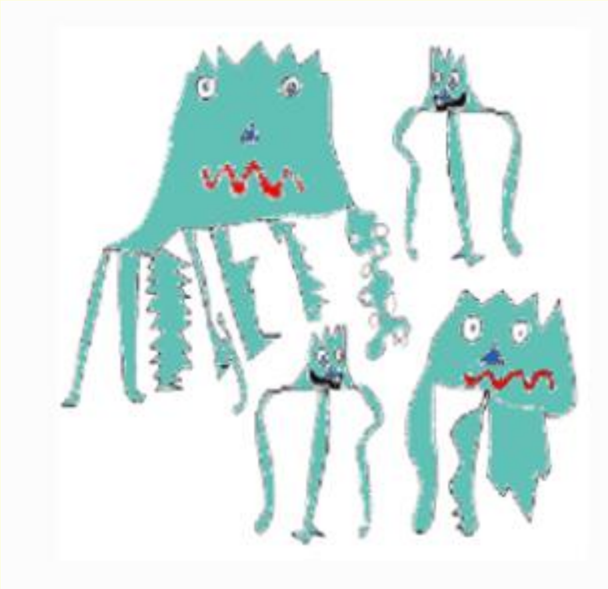
On the planet Vuv there are two sorts of creatures. The Zios have 3 legs and the Zepts have 7 legs.



The great planetary explorer Nico, who first discovered the planet, saw a crowd of Zios and Zepts. He managed to see that there was more than one of each kind of creature before they saw him. Suddenly they all rolled over onto their backs and put their legs in the air.

He counted 52 legs. How many Zios and how many Zepts were there?
Do you think there are any different answers?

Approaching the problem



Let’s start more simply. If Nico had seen 6 legs, what creatures might he have seen? What about 14 legs? What about 10 legs?

The children will then work with a partner on a whiteboard to solve these.

At this point, the children would share the different ways of recording.

The learning here is using multiples of 3 and 7.

We might scaffold this by using lolly sticks for legs.

We might challenge pupils to find out what other total numbers of legs Nico could have seen. Are there any numbers of legs he can’t have seen?

The food problem

Is there a more efficient way to solve this? Yes! But often children simply get stuck and give up. Trial and improvement can help in this situation as they can be successful.

Try this:

Zara bought a chicken burger and some fries for £2.85.

The chicken burger cost twice as much as the fries.

What was the cost of each one?

Using trial and improvement to solve the food problem

Zara bought a chicken burger and some fries for £2.85.

The chicken burger cost twice as much as the fries.

What was the cost of each one?

	Fries	Chicken burger	Total	Assessment
Guesstimate 1	£0.80	£1.60	£2.40	Too low
Guesstimate 2	£0.85	£1.70	£2.55	Too low
Guesstimate 3	£1.00	£2.00	£3.00	Too high
Guesstimate 4	£0.95	£1.90	£2.85	Found the answer!

The present problem

3 brothers save some money to buy their parents a present for Christmas. Between them, Max, Lennie and Harrison saved £20. Max saved £3 more than Lennie and Lennie saved £4 more than Harrison. How much did they each save?

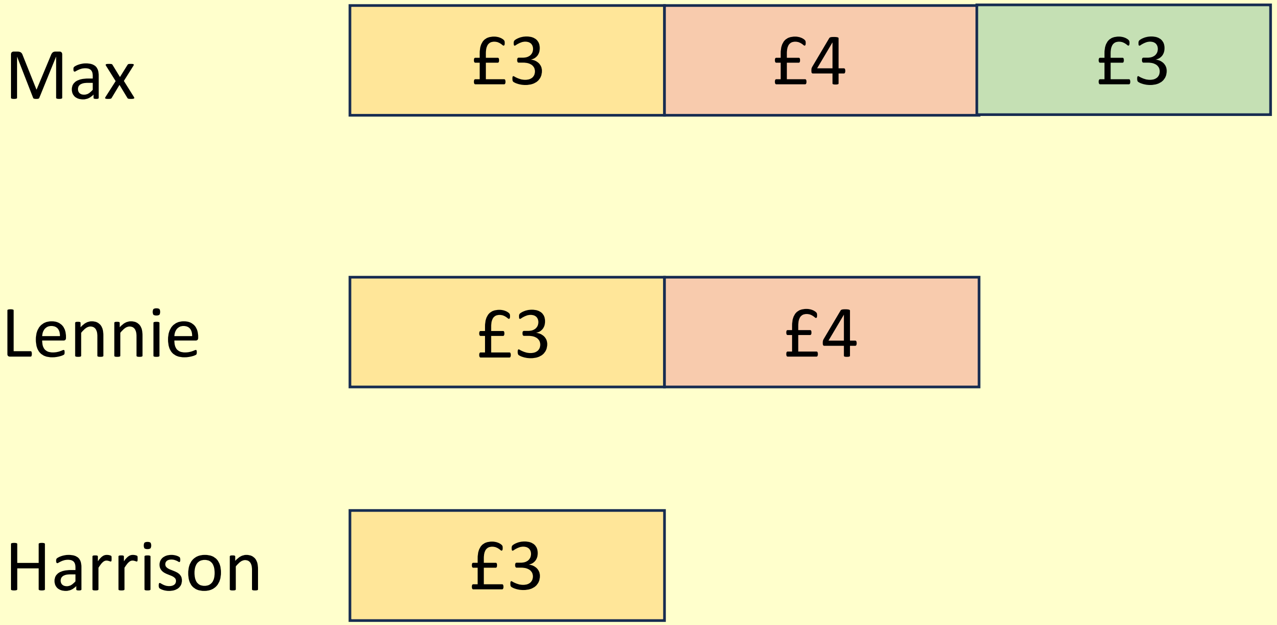
	Max	Lennie	Harrison	Total	Assessment
Guesstimate 1					
Guesstimate 2					
Guesstimate 3					
Guesstimate 4					

The bar model version

3 brothers save some money to buy their parents a present for Christmas.

Between them, Max, Lennie and Harrison saved £20.

Max saved £3 more than Lennie and Lennie saved £4 more than Harrison. How much did they each save?



Working systematically

In the context of problem solving, we could think about working systematically as working in a methodical and efficient way which could clearly show others that we are using a pattern or system.

A good place to start is with finding all the possibilities. This is an ideal way to become fluent with the skill. It’s important to teach the children a system where no possibilities are left out.

Working systematically- finding all possibilities

Here are 2 dice. If you add up the dots on top, you get a total of 7.

Roll 2 dice. Add the numbers that are on the top. What totals did you get?

What other totals could you get? Can you find all the possibilities?

1+1	2+2	3+3	4+4
1+2	2+3	3+4	4+5
1+3	2+4	3+5	4+6
1+4	2+5	3+6	
1+5	2+6		
1+6			

5+5	6+6
5+6	

Using logical reasoning

When reasoning logically, pupils are connecting information together in a sequence of steps. There is no guesswork involved. Tables and grids are an excellent tool to use as it helps to visualise the problem. Reading and understanding the clues are important. Pupils also need to decide where to begin as the clues may need to be managed in a different order to the order they appear.

5 children are talking about their favourite animals after a trip to the zoo. They are a puffin, alpaca, armadillo, giraffe and panda. Use the clues to work out who likes which animals.

Johnnie's favourite is a panda. Ani's favourite animals starts with the same letter. Nadia and Raul's favourite don't start with the same letter. Nadia likes alpacas best.

	Puffin	Alpaca	Panda	Giraffe	Armadillo
Ani	Y	x	x	x	x
Johnnie	x	x	Y	x	x
Nadia	x	Y	x	x	x
Raul	x	x	x	Y	x
Khoza	x	x	x	x	Y

5 children left their shoes in the hallway. Each child was a different size:
2, 3, 4, 5, 6

Tess knew that her shoes were the smallest. Kyle thought his were bigger than Charlie's but smaller than Deeptak's. Balroop knew his were the biggest.

Which size shoes does each child have?

	2	3	4	5	6
Tess					
Kyle					
Charlie					
Deeptak					
Balroop					

5 children left their shoes in the hallway. Each child was a different size:
2, 3, 4, 5, 6

Tess knew that her shoes were the smallest. Kyle thought his were bigger than Charlie's but smaller than Deeptak's. Balroop knew his were the biggest.
Which size shoes does each child have?

Charlie Kyle Deeptak

	2	3	4	5	6
Tess	Y	N	N	N	N
Kyle	N	N	Y	N	N
Charlie	N	Y	N	N	N
Deeptak	N	N	N	Y	N
Balroop	N	N	N	N	Y

Drawing a table

Tables help us to organise information so that it can be easily understood. Numbers and words can be made clearer and the relationship between them can become more obvious. A table can show part of the solution that it might be difficult to visualise. Deciding on the number of rows and columns and headings can be tricky!

The bike problem

Harry is doing a charity cycle ride. Each day he cycles less as he gets more tired. On the first day he cycles 38km, on the second 35km, and the third 32km and so on following the same pattern. How many days will it take him to cover a distance of 220km?

The bike problem

Harry is doing a charity cycle ride. Each day he cycles less as he gets more tired. On the first day he cycles 38km, on the second 35km, and the third 32km and so on following the same pattern. How many days will it take him to cover a distance of 220km?

Day	km cycled	Distance covered
1	38	38
2	35	
3	32	
4		
5		
6		
7		
8		

The bike problem

Day	km cycled	Distance covered
1	38	38
2	35	73
3	32	105
4	29	134
5	26	160
6	23	183
7	20	203
8	17	220

Working backwards

This particular strategy is useful for solving problems that include a sequence of events where some or part of the information has deliberately been omitted. In a problem with linked information, the events follow each other or a piece of information is influenced by what comes next.

The key to making this strategy work is for pupils to understand that whatever mathematical operations they come across will have to be reversed.

What was my number?

I am thinking of a number.

I subtract 10.

Then I divide by 7.

I halve this number.

Now I have 3.

What number was I thinking of?

What was my number?

I am thinking of a number.

I subtract 10.

Then I divide by 7.

I halve this number.

Now I have 3.

What number was I thinking of?

How to solve

I have 3

Previously I halved, so now I need to double (6)

Previously I divided by 7, so now I need to multiply by 7 (42)

Previously, I subtracted 10, so now I need to add 10 (52)

The number I was thinking of was 52

The weighty problem

4 gorillas were weighed. Clint was 15kg lighter than Amish. Gibbo was twice as heavy as Clint. Jimmy was 7kg heavier than Gibbo. If Jimmy weighed 71kg, what was Amish's weight?

The weighty problem

4 gorillas were weighed. Clint was 15kg lighter than Amish. Gibbo was twice as heavy as Clint. Jimmy was 7kg heavier than Gibbo. If Jimmy weighed 71kg, what was Amish's weight?

We know that Jimmy weighed 71kg

Jimmy was 7kg heavier than Gibbo, so we need to subtract 7 from 71 to work out Gibbo's weight (64kg)

We know that Gibbo was twice as heavy as Clint, so Clint must be half of 64kg (32kg)

We know that Clint was 15kg lighter than Amish so we need to add 15 to 32 to work out Amish's weight (47kg)

Clint	32kg
Amish	47kg
Gibbo	64kg
Jimmy	71kg

Prove it

5, 6 and 7 are consecutive numbers. They add up to 18.

14, 15 and 16 are consecutive numbers. They add up to 45.

Take other sets of 3 consecutive numbers.

What do you notice?

Do the totals have anything in common?

How can you be sure that what you have noticed will always be true?

How many examples do you need to prove it is always true?

Is there a way to know for sure that it will always be true?

Prove it

5, 6 and 7 are consecutive numbers. They add up to 18.

14, 15 and 16 are consecutive numbers. They add up to 45.

They all add up to create multiples of 3.

But how do we know that this will always be true?

We know because adding 3 consecutive numbers is the same as adding the middle number 3 times or multiplying it by 3. We therefore know they will always add up to be a multiple of 3.

Conjecturing and generalising

Always, Sometimes or Never true?

Questions to consider

Can you think of an example when it isn't true?
How do you know that it is always true?
Is it possible to check all examples? Is there another way of knowing?

The sum of 3 numbers is odd

If you add 1 to an even number, you get an odd number

Multiples of 5 end in a 5

If you add two odd numbers, you get an odd number

If you add a multiple of 10 to a multiple of 5, you get a multiple of 5

When you multiply 2 numbers, you will always get a bigger number

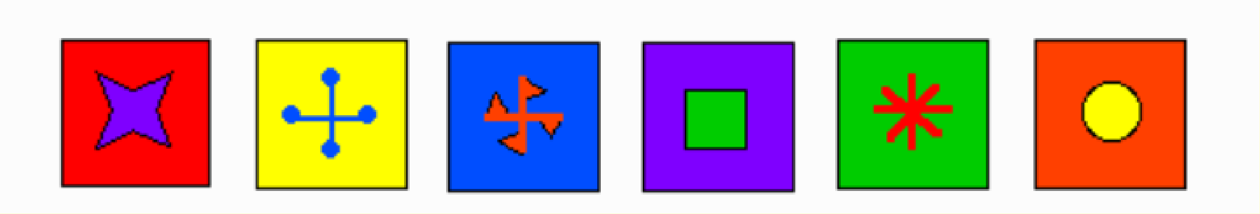
If you add a number to 5, your answer will be bigger than 5

A square number has an even number of factors

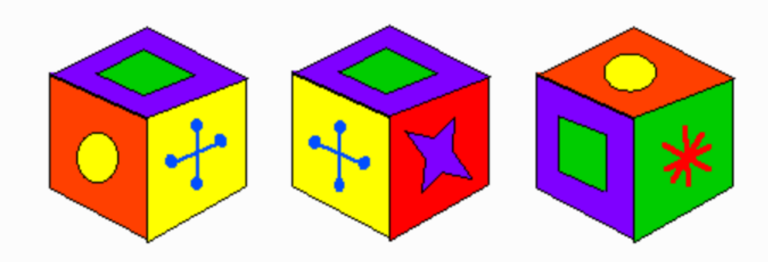
The sum of 3 consecutive numbers are divisible by 3

Dividing a whole number by a half makes it twice as big

Visualising



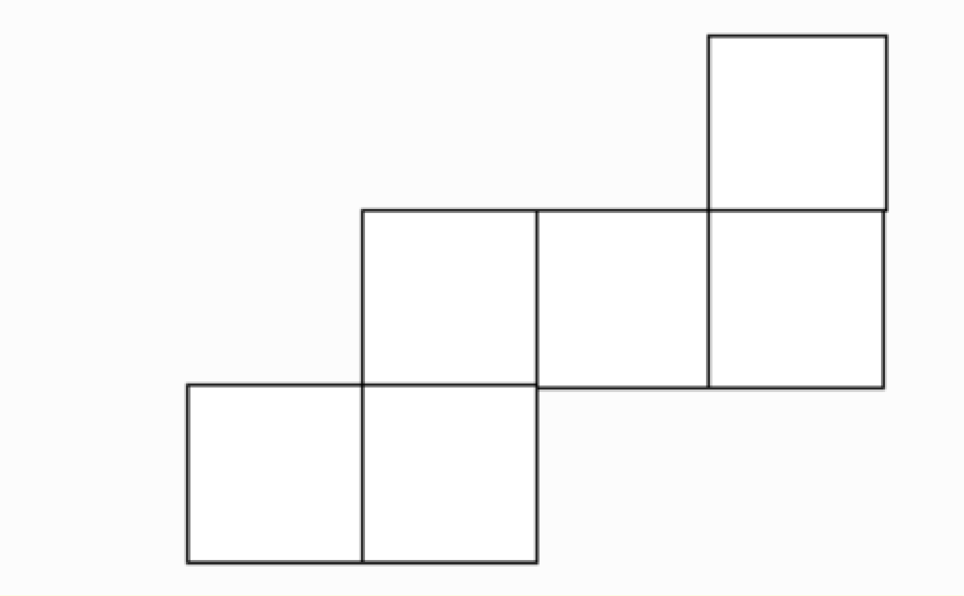
Here are the 6 faces of a cube



Here are 3 views of the cube

Can you work out where the faces are in relation

To one another and record them on the net?

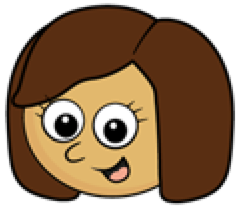


Brilliant Bar Models

Bar models are amazing! They are a way of visualising maths problems and abstract concepts. Sometimes, a problem seems too difficult- until you being to draw the bars. They help us to solve problems in a wide range of maths topics.

Kim and Eva have 60 sweets in total.

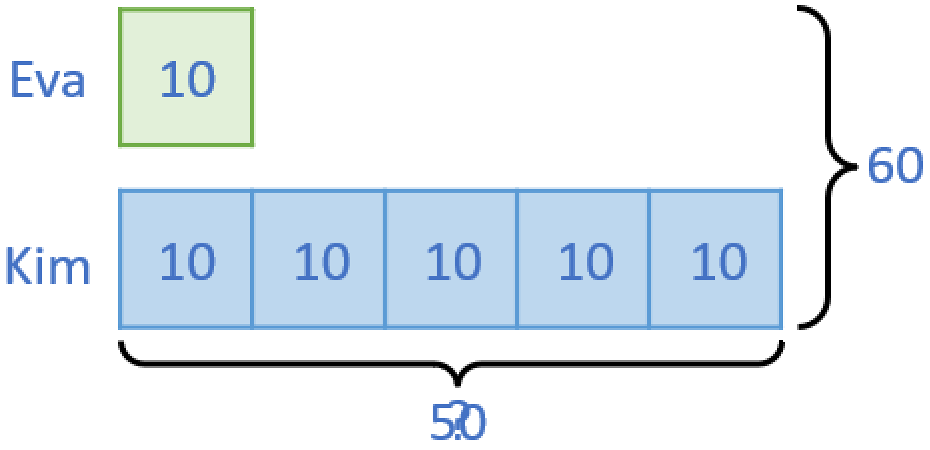
I have 5 times as many sweets as Eva.



Kim

How many sweets does Kim have?

Model



Calculations

$60 \div 6 = 10$

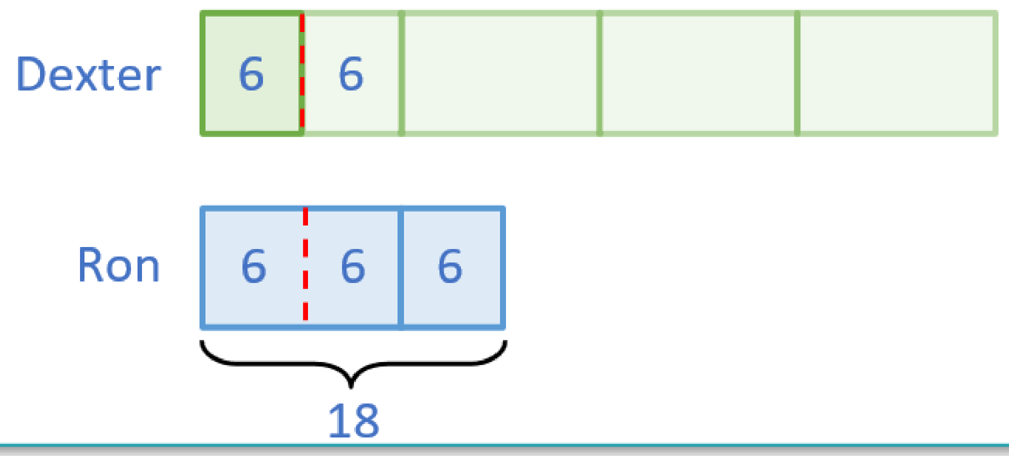
$10 \times 5 = 50$

Kim has __50__ sweets.

Dexter has 4 times as many toys as Ron.

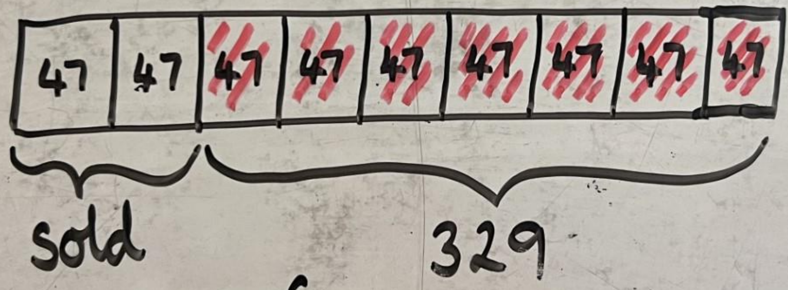
- He gives $\frac{3}{4}$ of his toys to charity.
- He gives $\frac{1}{2}$ of his remaining toys to Ron.
- Dexter has 6 toys left.

How many toys does Ron have now?

Model	**Calculations**
	$6 \times 3 = 18$

Ron now has __18__ toys.

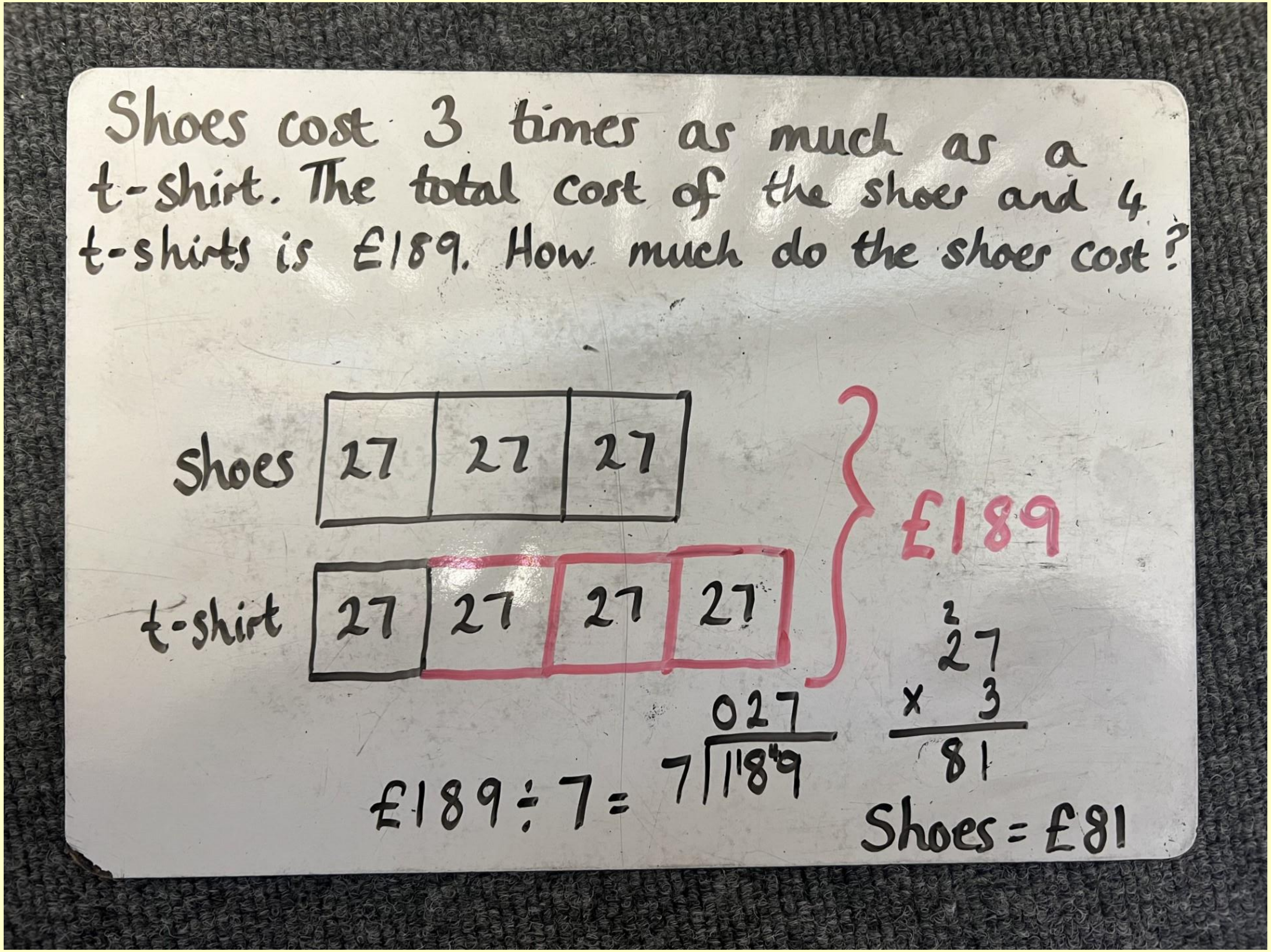
A shop has some pens. $\frac{2}{9}$ of the pens are sold. There are 329 pens left. How many pens did the shop have to begin with?



$$7 \overline{)3^{3}2^{4}9} = 047$$

$$\begin{array}{r} {}^{6}\\ 47 \\ \times \quad 9 \\ \hline 423 \end{array}$$

423 pens


Shoes cost 3 times as much as a t-shirt. The total cost of the shoes and 4 t-shirts is £189. How much do the shoes cost?
shoes
27
27
27
t-shirt
27
27
27
27
£189
027
7 | 189
£189 ÷ 7 =
27
x 3
81
Shoes = £81